Polynomials non-negative on non-compact semialgebraic sets

Abstract: Recently, M. Marshall answered a long-standing question in real algebraic geometry by showing that if $f(x, y) \in \mathbb{R}[x, y]$ and $f(x, y) \geq 0$ on the strip $[0, 1] \times \mathbb{R}$, then f has a representation $f = \sigma_0 + \sigma_1 x(1 - x)$, where $\sigma_0, \sigma_1 \in \mathbb{R}[x, y]$ are sums of squares.

In this talk, we give the background to this result, which goes back to Hilbert's 17th problem, and our generalizations to other non-compact basic closed semialgebraic sets of $\mathbb{R}^2$ which are contained in strip. We also give some negative results.