

MATHEMATICAL PRINCIPLES
OF NATURAL PHILOSOPHY

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x PREFACE TO THE FIRST EDITION

SINCE the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things, and the moderns, rejecting substantial forms and occult qualities, have endeavored to subject the phenomena of nature to the laws of mathematics, I have in this treatise cultivated mathematics as far as it relates to philosophy. The ancients considered mechanics in a twofold respect; as rational, which proceeds accurately by demonstration, and practical. To practical mechanics all the manual arts belong, from which mechanics took its name. But as artificers do not work with perfect accuracy, it comes to pass that mechanics is so distinguished from geometry that what is perfectly accurate is called geometrical; what is less so, is called mechanical. However, the errors are not in the art, but in the artificers. He that works with less accuracy is an imperfect mechanic; and if any could work with perfect accuracy, he would be the most perfect mechanic of all, for the description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn, for it requires that the learner should first be taught to describe these accurately before he enters upon geometry, then it shows how by these operations problems may be solved. To describe right lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics, and by geometry the use of them, when so solved, is shown; and it is the glory of geometry that from those few principles, brought from without, it is able to produce so many things. Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring. But since the manual arts are chiefly employed in the moving of bodies, it happens that geometry is commonly referred to their magnitude, and mechanics to their motion. In this sense rational mechanics will be the science of motions resulting from any forces whatsoever, and of the forces required to produce any motions, accurately proposed and demonstrated. This part of mechanics, as far as it extended to the five powers which relate to manual arts, was cultivated by the ancients, who considered gravity (it not being a manual power) no otherwise than in moving weights by those powers. But I consider philosophy rather than arts and write not concerning manual but natural powers, and consider chiefly those things which relate to gravity, levity, elastic force, the resistance of fluids, and the like forces, whether attractive or impulsive; and therefore I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; and to this end the general propositions in the first and second books are directed. In the third book I give an example of this in the explication of the System of the World; for by the propositions mathematically demonstrated in the former books in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and

the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea. I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are either mutually impelled towards one another, and cohere in regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of philosophy.

In the publication of this work the most acute and universally learned Mr. Edmund Halley not only assisted me in correcting the errors of the press and preparing the geometrical figures, but it was through his solicitations that it came to be published; for when he had obtained of me my demonstrations of the figure of the celestial orbits, he continually pressed me to communicate the same to the Royal Society, who afterwards, by their kind encouragement and entreaties, engaged me to think of publishing them. But after I had begun to consider the inequalities of the lunar motions, and had entered upon some other things relating to the laws and measures of gravity and other forces; and the figures that would be described by bodies attracted according to given laws; and the motion of several bodies moving among themselves; the motion of bodies in resisting mediums; the forces, densities, and motions, of mediums; the orbits of the comets, and such like, I deferred that publication till I had made a search into those matters, and could put forth the whole together. What relates to the lunar motions (being imperfect), I have put all together in the corollaries of Prop. 66, to avoid being obliged to propose and distinctly demonstrate the several things there contained in a method more prolix than the subject deserved and interrupt the series of the other propositions. Some things, found out after the rest, I chose to insert in places less suitable, rather than change the number of the propositions and the citations. I heartily beg that what I have here done may be read with forbearance; and that my labors in a subject so difficult may be examined, not so much with the view to censure, as to remedy their defects.

IS. NEWTON

Cambridge, Trinity College, *May 8, 1686*

PREFACE TO THE SECOND EDITION

In this second edition of the *Principia* there are many emendations and some additions. In the second section of the first book, the determination of forces, by which bodies may be made to revolve in given orbits, is illustrated and enlarged. In the seventh section of the second book the theory of the resistances of fluids was more accurately investigated, and confirmed by new experiments. In the third book the lunar theory and the precession of the equinoxes were more fully deduced from their principles; and the theory of the comets was

confirmed by more examples of the calculation of their orbits, done also with greater accuracy.

IS. NEWTON

London, *March* 28, 1713

PREFACE TO THE THIRD EDITION

IN this third edition, prepared with much care by Henry Pemberton, M.D., a man of the greatest skill in these matters, some things in the second book on the resistance of mediums are somewhat more comprehensively handled than before, and new experiments on the resistance of heavy bodies falling in air are added. In the third book, the argument to prove that the moon is retained in its orbit by the force of gravity is more fully stated; and there are added new observations made by Mr. Pound, concerning the ratio of the diameters of Jupiter to one another. Some observations are also added on the comet which appeared in the year 1680, made in Germany in the month of November by Mr. Kirk; which have lately come to my hands. By the help of these it becomes apparent how nearly parabolic orbits represent the motions of comets. The orbit of that comet is determined somewhat more accurately than before, by the computation of Dr. Halley, in an ellipse. And it is shown that, in this elliptic orbit, the comet took its course through the nine signs of the heavens, with as much accuracy as the planets move in the elliptic orbits given in astronomy. The orbit of the comet which appeared in the year 1723 is also added, computed by Mr. Bradley, Professor of Astronomy at Oxford.

IS. NEWTON

London, *Jan.* 12, 1725-6