

## PROPOSITION 69. THEOREM 29

*In a system of several bodies A, B, C, D, &c., if any one of those bodies, as A, attract all the rest, B, C, D, &c., with accelerative forces that are inversely as the squares of the distances from the attracting body; and another body, as B, attracts also the rest, A, C, D, &c., with forces that are inversely as the squares of the distances from the attracting body; the absolute forces of the attracting bodies A and B will be to each other as those very bodies A and B to which those forces belong.*

For the accelerative attractions of all the bodies B, C, D, towards A, are by the supposition equal to each other at equal distances; and in like manner the accelerative attractions of all the bodies towards B are also equal to each other at equal distances. But the absolute attractive force of the body A is to the absolute attractive force of the body B as the accelerative attraction of all the bodies towards A is to the accelerative attraction of all the bodies towards B at equal distances; and so is also the accelerative attraction of the body B towards A to the accelerative attraction of the body A towards B. But the accelerative attraction of the body B towards A is to the accelerative attraction of the body A towards B as the mass of the body A is to the mass of the body B; because the motive forces which (by the second, seventh and eighth Definitions) are as the accelerative forces and the bodies attracted conjointly are here equal to one another by the third Law. Therefore the absolute attractive force of the body A is to the absolute attractive force of the body B as the mass of the body A is to the mass of the body B. Q.E.D.

COR. I. Therefore if each of the bodies of the system A, B, C, D, &c., does singly attract all the rest with accelerative forces that are inversely as the squares of the distances from the attracting body, the absolute forces of all those bodies will be to each other as the bodies themselves.

COR. II. By a like reasoning, if each of the bodies of the system A, B, C, D, &c., does singly attract all the rest with accelerative forces, which are either inversely or directly in the ratio of any power whatever of the distances from the attracting body; or which are defined by the distances from each of the attracting bodies according to any common law; it is plain that the absolute forces of those bodies are as the bodies themselves.

COR. III. In a system of bodies whose forces decrease as the square of the distances, if the lesser revolve about one very great one in ellipses, having their common focus in the centre of that great body, and of a figure exceedingly accurate; and moreover by radii drawn to that great body describe areas proportional to the times exactly; the absolute forces of those bodies to each other will be either accurately or very nearly in the ratio of the bodies. And so conversely. This appears from Cor. of Prop. 68, compared with the first Corollary of this Proposition.

## SCHOLIUM

These Propositions naturally lead us to the analogy there is between centripetal forces and the central bodies to which those forces are usually directed; for it is reasonable to suppose that forces which are directed to bodies should depend upon the nature and quantity of those bodies, as we see they do in magnetical experiments. And when such cases occur, we are to compute the attractions of the bodies by assigning to each of their particles its proper force, and then finding the sum of them all. I here use the word *attraction* in general

for any endeavor whatever, made by bodies to approach to each other, whether that endeavor arise from the action of the bodies themselves, as tending to each other or agitating each other by spirits emitted; or whether it arises from the action of the ether or of the air, or of any medium whatever, whether corporeal or incorporeal, in any manner impelling bodies placed therein towards each other. In the same general sense I use the word *impulse*, not defining in this treatise the species or physical qualities of forces, but investigating the quantities and mathematical proportions of them; as I observed before in the Definitions. In mathematics we are to investigate the quantities of forces with their proportions consequent upon any conditions supposed; then, when we enter upon physics, we compare those proportions with the phenomena of Nature, that we may know what conditions of those forces answer to the several kinds of attractive bodies. And this preparation being made, we argue more safely concerning the physical species, causes, and proportions of the forces. Let us see, then, with what forces spherical bodies consisting of particles endued with attractive powers in the manner above spoken of must act upon one another; and what kind of motions will follow from them.

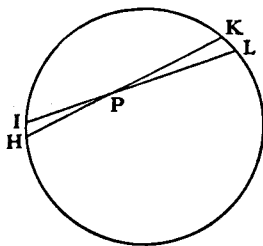
## SECTION XII

### THE ATTRACTIVE FORCES OF SPHERICAL BODIES

#### PROPOSITION 70. THEOREM 30

*If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from those points, I say, that a corpuscle placed within that surface will not be attracted by those forces any way.*

Let HIKL be that spherical surface, and P a corpuscle placed within. Through P let there be drawn to this surface two lines HK, IL, intercepting very small arcs HI, KL; and because (by Cor. III, Lem. 7) the triangles HPI,



LPK are alike, those arcs will be proportional to the distances HP, LP; and any particles at HI and KL of the spherical surface, terminated by right lines passing through P, will be as the square of those distances. Therefore the forces of these particles exerted upon the body P are equal between themselves. For the forces are directly as the particles, and inversely as the square of the distances. And these two ratios compose the ratio of equality, 1 : 1. The attractions therefore, being equal, but exerted in opposite directions, de-

stroy each other. And by a like reasoning all the attractions through the whole spherical surface are destroyed by contrary attractions. Therefore the body P will not be any way impelled by those attractions. Q.E.D.

#### PROPOSITION 71. THEOREM 31

*The same things supposed as above, I say, that a corpuscle placed without the spherical surface is attracted towards the centre of the sphere with a force inversely proportional to the square of its distance from that centre.*

Let AHKB, *ahkb* be two equal spherical surfaces described about the centres S, s; their diameters AB, *ab*; and let P and *p* be two corpuscles situate without