Math/Phil 270

History and Philosophy of Mathematics

Fall 2008, Tue-Th 1:00–2:15

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Contents: Topics in the history of mathematics and their philosophical background. Genesis and evolution of ideas in analysis, algebra, geometry, mechanics, foundations. Historical and philosophical aspects of concepts of infinity, mathematical rigor, probability, etc. The emergence of mathematical schools.

Particulars: In this course we will learn about the emergence of the Calculus in the 17th and 18th centuries, with an emphasis on early modern authors. In particular, we will study how the basic concepts (functions, continuity, limits, derivatives, integrals, etc.) were developed, and what were the motivations of the mathematicians, scientists, and philosophers who laid the foundations of the Calculus.

Course requirements:

- *Timely and thoughtful completion of the readings assigned for class.* Preparation for participation in class is required. Participation is understood to include responding to questions, asking questions, and attentively listening to others. Careful reading should enable you to answer to questions in class.

- *Regular class attendance and thoughtful contribution to the class discussion.* We will take roll at the beginning of class. Students who are counted as absent for more than 4 classes will receive a 1/3 grade deduction on their final course grade (i.e., an A- will become a B+, a C will become a C-, etc.). Another 1/3 grade deduction will occur when a student has been counted absent for more than 7 classes. If you are counted absent for more than 10 classes, you will not pass the course.

- *Four essays of approximately seven pages.* These essays are due on the dates listed in the attached reading schedule. Late essays will not be accepted except in the most extraordinary circumstances, unless arrangements have been made with the instructor prior to the due date of the assignment. Two of the first three essays may be revised in light of the instructors' criticisms, and you are encouraged to do so. The revised essay is due two weeks from the date that the essay is handed back. You should discuss the criticism with one of your instructors before you start to revise the essay. The final grade will take the original one into account.

- *Summaries.* At the beginning of each class, one student (chosen at random) will be asked to summarize the contents of the previous class.

Grading:

The essays count 20% each, class participation and thoughtful contribution another 20%.

Honor Code:

You should be aware of the significance and validity of the Emory College Honor Code:

http://www.college.emory.edu/current/standards/honor_code.html

If you use books or articles or the Internet for writing your essays you are obliged to name your sources exactly in the footnotes. If you use a text verbatim in your own work you have to put the passage into quotation marks.

Prerequisites:

MATH 112, 112Z, 112S or permission of the instructor.

Please note:

1. The attached schedule is tentative and may be revised in light of developments during the semester. Any changes will be announced in class.
2. Any student who needs special accommodation for a disability should inform the instructors of these needs during the first week of class.
3. If you have to miss a class because of a serious reason, it will count as excused only when you have informed one of the instructors (e.g., by e-mail).

Text book:

Ivor Grattan–Guinness, *The Rainbow of Mathematics: A History of the Mathematical Sciences*, W. W. Norton and Co., New York, 2000.

As a general resource for biographies of mathematicians and history of mathematics, we recommend St. Andrews's University *Mac Tutor History of Mathematics Archive*:

http://www-groups.dcs.st-and.ac.uk/history/index.html

Excerpts from the following sources will be distributed in class or posted electronically on the class web site:

GALILEO GALILEI, *The Assayer* (1623). In: *Discoveries and Opinions of Galileo*. Translated with an Introduction and Notes by Stillman Drake. Random House, New York, NY, 1957.
— *Dialogues Concerning Two New Sciences* (1638). Translated by Henry Crew and Alfonso de Salvio, with an Introduction by Antonio Favaro. Dover Publications, Inc., New York, NY, 1954.

RENÉ DESCARTES, *Rules for Guiding One's Intelligence in Searching for the Truth* (1628). In *Discourse on Method and Related Writings*. Translated and with an Introduction by Desmond M. Clarke, Penguin Classics, London, 1999.

—, *The Geometry* (1637). Translated by David E. Smith and Marcia L. Latham, Thirtieth printing, Encyclopaedia Britannica, Inc., Chicago, IL, 1988.

Pierre de Fermat, *On a Method for the Evaluation of Maxima and Minima* (1637). Translated by D. J. Struik, in *Classics of Mathematics*, Edited by Ronald Calinger, Moore Publishing Company, Inc., Oak Park, IL, 1982.

Blaise Pascal, *The Spirit of Geometry* (1662), from the Introduction to the *Logic of Port Royal*, by Antoine Arnauld and Pierre Nicole, available online (see link from course web page).

—, *The Wager*, no. 233 from the *Pensées* (ca. 1657). Available online at
http://www.stat.ucla.edu/history/pascal_wager.pdf

Thomas Hobbes, *Leviathan* (1655). Revised student edition. Edited by Richard Tuck, Cambridge University Press, Cambridge, 1996.

Baruch Spinoza, *The Letters* (ca. 1660-70). Translated by Samuel Shirley, Introduction and Notes by Steven Barbone, Lee Rice, and Jacob Adler, Hackett Publishing Co., Inc. Indianapolis/Cambridge, 1995.

John Locke, *An Essay Concerning Human Understanding* (1690). Edited with a Foreword by Peter H. Nidditch, Oxford University Press, 1987.

Gottfried Wilhelm Leibniz, *New Essays on Human Understanding* (1704). Translated and edited by Peter Remnant and Jonathan Bennett, Cambridge University Press, 1981.

—, *A New Method for Maxima and Minima as Well as Tangents, Which is Impeded Neither by Fractional nor by Irrational Quantities, and a Remarkable Type of Calculus for This* (1684). Translated by D. J. Struik, in *Classics of Mathematics*.

—, From *Supplementum Geometriae Dimensoriae...* (1693). Ibidem.

—, *Discourse on Metaphysics* (1686). Edited and Translated by Daniel Garber and Roger Ariew, Hackett, Indianapolis, IN, 1991.

John Wallis, *The Arithmetic of Infinitesimals* (1655). Translation by Jacqueline A. Stedall, Springer, New-York, 2004.

Isaac Newton, *Mathematical Principles of Natural Philosophy* (1687; 3rd Ed., 1726).

—, *Letters to Henry Oldenburg* (1676), on the binomial series and on his general method of finding quadratures.

—, From the Introduction to the *Tractatus de Quadratura Curvarum* (1704).

—, *Opticks* (1704; 2nd Ed., 1717).

Johann Bernoulli, *The Curvature of a Ray in Nonuniform Media* (on the Brachistochrone) (1697). Translated by D. J. Struik, in *Classics of Mathematics*.,

Pierre Moreau de Maupertuis, *Accord Between Different Laws of Nature that Seemed Incompatible* (On the Principle of Least Action) (1744). Anonymous English translation available through Wikipedia.

Jean Le Rond d'Alembert, *Differential* (on Limits), from *Encyclopédie, Vol. 4* (1754). Translated by D. J. Struik, in *Classics of Mathematics*.

Schedule:

- Th, Aug 28: Introduction.
- Tue, Sep 2: **Galileo Galilei**. Galileo and the mathematization of the world. Read Grattan-Guiness, pp. 210–216 and Galileo's *The Assayer*, pp. 237–238 (see course webpage). What does Galileo mean by the 'book of nature' and how it is written in figures and numbers? Read Galileo's *Dialogues*, pp. 1–6. What does he say about the relation of geometry and mechanics?
- Th, Sep 4: Galileo's discovery of the law of free falling bodies and his use of 'pre-calculus' techniques. Read p. 62 of the *Dialogues*. We will discuss the relation to his philosophical critique of Aristotle's opinion about the vacuum. You should start the reading for the next two classes!
- Tue, Sep 9: Galileo on infinitely large and infinitely small quantities (indivisibles). Read *Dialogues*, pp. 20–49 (you may skip the more technical passages). What is Galileo's understanding of the relation between finite quantities and the infinite? What does he mean by the 'single stroke' on p. 48? We will also discuss the Greek's position towards the infinite.
- Th, Sep 11: Galileo's atomism. Read *The Assayer*, pp. 274-279 and the *Dialogues*, pp. 7–20 and 25–44. What is his opinion about the cohesion and adhesion of bodies? How are bodies composed? We will discuss the Greek atomistic position and compare it with that of Galileo. How is the vacuum related to the problem of indivisibles in mathematics on the one side and to atomistic philosophy on the other side? Read the *Dialogues*, pp. 48–52 and pp. 60–61.
- Tue, Sep 16: Read Galileo, *The Assayer*, pp. 266–279. What does Galileo mean by 'demonstration'? What are Galileo's methodological demands to obtain certain knowledge? Try to understand his strict methodology as a starting point of early modern philosophy, as well as of modern science. Why he is so annoyed by the 'whatever else' of his critic (p. 273)?
- Th, Sep 18: **René Descartes** and his new approach to philosophy and geometry. Read Grattan–Guinness, pp. 223–226. The impact of the new mechanics on traditional geometry. The certainty of mathematics. The *mathesis universalis*. Read from the *Rules*, pp. 117–130 (rules 1–4).
- Tue, Sep 23: **First paper due**. The creation of analytic geometry. Read Descartes, *Geometry*, pp. 295–308 (don't worry about the technical discussion of the problem of Pappus). Can you find the notion of *function* on p. 303?
- Th, Sep 25: **Pierre de Fermat** and his contributions to the creation of analytic geometry and the calculus. Read from *On a Method for the Evaluation of Maxima and Minima*, pp. 339–340.
- Tue, Sep 30: The use of the geometrical method in philosophy; the role of **definitions**. Read **Pascal**, *On Geometrical Spirit*, pp. 385–387; **Hobbes**, from the *Leviathan*, pp. 24–31; **Spinoza**, the letter on definition, pp. 91–92.
- Th, Oct 2: The emergence of **probability**: correspondence between Fermat and Pascal. Read Grattan–Guinness, pp. 284–287. Mathematical examples of calculating chances. Binomial coefficients and the arithmetic triangle.

- Tue, Oct 7: The philosophical discussion on probability. Read Spinoza's letter, p. 278; read **Locke** on probability, from the *Essay on Human Understanding*, pp. 654–657; read Leibniz on probability, from the *New Essays on Human Understanding*, pp. 457–459. We will discuss the turn from Descartes's strict demand for mere certainty in science toward a higher regard of probable knowledge in science—obtained, however, by a rigorous method.

- Th, Oct 9: Probability versus certainty. Knowledge of probability as a way to rely on God. Read Pascal's *Wager*, pp. 1–3 at: http://www.stat.ucla.edu/history/pascal_wager.pdf
  We will discuss the significance of some uncertainty of knowledge for the defense of Christian religion.

- Tue, Oct 14: **Fall Break** (no class).

- Th, Oct 16: "Pre-calculus." The new approach of **Kepler**, **Cavalieri**, **Torricelli**, **Wallis**. Infinite series and products. Read Grattan-Guinness, pp. 203–210 and pp. 227–228.

- Tue, Oct 21: Thomas Hobbes' foundation of a materialistic geometry and his controversy with Wallis. The influence of Italian mathematics on Hobbes' approach to geometry and his foundation of geometry on mechanical motion. Hobbes' notion of *conatus* as a "pre-calculus" concept. You may want to start the reading assignment for next time!

- Th, Oct 23: **Isaac Newton**. Biographical notes. Newton's development of the Calculus, part I. The Binomial Theorem. Read Grattan-Guinness, pp. 234–253. Read Newton's two *Letters to Oldenburg* from 1676.

- Tue, Oct 28: Newton's development of the Calculus, part II. Read Grattan-Guinness, pp. 253–268. Read the passages from Newton's *Mathematical Principles of Natural Philosophy* on "prime and ultimate ratios" and on "moments."

- Th, Oct 30: **Second paper due**. Newton's development of the Calculus, part III. Read from the Introduction to Newton's *Tractatus de Quadratura Curvarum* (Treatise on the Quadrature of Curves).

- Tue, Nov 4: Read the assigned texts from the *Principia*. We will try to discover Newton's secret or hidden "Philosophy" in his "Mathematical Principles." Try to answer the following questions: What is his method (in comparison with that of Galileo and Descartes)? Why does he reject hypotheses? What does Newton say about gravity? What is action at a distance?

- Th, Nov 6: Read Grattan–Guinness, pp. 272–274 and the assigned passages from the *Optics*. We will continue the discussion of Newton's hidden philosophy. The lecture will examine Newton's studies of alchemy, theology, the ancients, and the Old Testament. What kinds of forces are discussed by Newton? What does he say about the absolute hardness of bodies? What about active principles? Can you see the Design theory in his argument?

- Tue, Nov 11: We will finish the discussion of Newton's hidden philosophy and begin the session on **Leibniz** with an introduction to his biography, his philosophy and his turn to mathematics as initiated by his study of Hobbes and encouraged by Huygens.

- Th, Nov 13: Leibniz' development of the Calculus, part I. Read the assigned text on the Differential Calculus (*A New Method for Maxima and Minima as Well as Tangents, ...*, pp. 348–354.) Leibniz' proof of the Law of Refraction in Optics.

- Tue, Nov 18: Leibniz' development of the Calculus, part II. Read the assigned text on the Fundamental Theorem of Calculus (*Supplementum Geometriae Dimensoriae ...*, pp. 355–356.) Remarks on other contributions of Leibniz to logic, mathematics, mechanics, etc. The Newton–Leibniz controversy.

- Th, Nov 20: Read the paragraphs 17 and 18 of Leibniz' Discourse on Metaphysics. We will discuss Leibniz' distinction of "motion" and "force," motion being relative whereas force is absolute and necessary for the explanation of motion. Leibniz uses this as his justification of his view that kinematics is founded on geometry while dynamics is founded on metaphysics. We will discuss how this is explanatory for his metaphysics.

- Tue, Nov 25: Read the paragraphs 19, 21, 22 in the same text. Notice Leibniz' reintroduction of final causes and his reconciliation with efficient causes. How is this different from Newton's reintroduction of final causes in terms of their use in science, i.e., physics vs. metaphysics? Why are the final causes so important for Newton and Leibniz? **Third paper due**.

- Th, Nov 27: **Thanksgiving** (no class).

- Tue, Dec 2: The development of the Calculus by the **Bernoullis**, **de l'Hôpital**, and **Euler**. The Calculus of Variations (isoperimetric problems, the brachistochrone). Read the assigned text by Johann Bernoulli. Read Grattan-Guinness, pp. 278–280, 293–308.

- Th, Dec 4: Read Grattan-Guinness, pp. 315–317 (on variational mechanics) and the assigned text by **Maupertuis** (on the Principle of Least Action). Maupertuis, König, Euler, Voltaire and the controversy on the Principle of Least Action.

- Tue, Dec 9: Read the assigned text by **d'Alembert** (on limits as the foundation of the Calculus). Summary and concluding discussion.

- Fri, Dec 12, 4:30-7:00: **Final paper due**.